The bisection method

1. Use three steps of the bisection method to approximate a root of the function $f(x) \overset{\text{def}}{=} \frac{\sin(x)}{x} + e^{-x}$ starting with $a_0 = 3.0$ and $b_0 = 4.0$.

Answer: To ten significant digits, we have [3, 4], [3, 3.5], [3.25, 3.5], [3.25, 3.375]

2. Given that the maximum absolute error at one step of the bisection method is $b_k - a_k$, demonstrate that the maximum absolute error after the next step is $(b_k - a_k)/2$.

Answer: See the course notes.

3. Use three steps of the bisection method to approximate a root of the function $f(x) \overset{\text{def}}{=} x^3 - 3x + 1$ starting with $a_0 = 1$ and $b_0 = 2$.

Answer: [1, 2], [1.5, 2], [1.5, 1.75], [1.5, 1.625].

4. If you continue to iterate the bisection method in Question 3, what root does it converge to?

Answer: To ten significant digits, 1.532088886

5. In general, should you apply the bisection method if you don't already have an idea as to what a root of a function is?

Answer: The answer is perhaps. For the bisection method, you must already have a bracket of the root, although the function may also have a discontinuity instead of a root.

6. The function x^2 has a double root at $x = 0$. Can you apply the bisection method to find a double root?

Answer: No, for the function has the same sign in the vicinity of a root, so it is impossible to bracket it. Also, whereas x^2 has a double root at $x = 0$, the function $x^2 + \varepsilon$ has no real roots for $\varepsilon > 0$.